

### III.C.1.c. Resolution broadening: convolution of Gaussian and exponential

Gaussian and exponential resolution broadening may be invoked simultaneously, giving the broadened cross section (or transmission) as

$$f_{GE}(E) = \frac{1}{\Delta_E \Delta_G \pi} \int_{E-\Delta E_s}^{\infty} dE'' \exp \left\{ -\frac{(E'' - (E - \Delta E_s))^2}{\Delta_E^2} \right\} \times \int_{-\infty}^{+\infty} dE' \exp \left\{ -\frac{(E' - E'')^2}{\Delta_G^2} \right\} f(E') , \quad (\text{III C1 c.1})$$

where the shift  $\Delta E_s$  is introduced in order that the maximum of the broadening function be located at  $E = E'$ .

Rearrangement of the integrands in Eq. (III C1 c.1) and explicit integration over the  $E''$  variable yield

$$f_{GE}(E) = \frac{1}{2\Delta_E} \exp \left\{ \frac{\Delta_G^2}{4\Delta_E^2} \right\} \int_{-\infty}^{+\infty} dE' f(E') \exp \left\{ -\frac{(E' - E + \Delta E_s)^2}{\Delta_E^2} \right\} \times \text{erfc} \left( \frac{\Delta_G}{2\Delta_E} - \frac{(E' - E + \Delta E_s)}{\Delta_G} \right) . \quad (\text{III C1 c.2})$$

Historical Note: this is not the form which is given in the MULTI manual [GA74] but is the (correct) form that is used in both the MULTI and the SAMMY codes.

The energy shift  $\Delta E_s$  is found by setting  $g(E') = \exp(\cdot) \text{erfc}(\cdot)$  from the integrand of Eq. (III C1 c.2) and locating the value of  $E'_{\max}$  for which  $g(E'_{\max})$  is a maximum by setting  $dg/dE' = 0$  at that value. This value  $E'_{\max}$  is then set equal to  $E$ , and the resulting equation solved for  $\Delta E_s$ . Newton's method is used to find the solution.

Alternatively, one can assume that the energy shift is zero. This requires the inclusion of a line in the INPut file reading

DO NOT SHIFT ENERGY for exponential tail on resolution broadening

The lower and upper integration limits in Eq. (III C1 c.2) are truncated to  $E - 5\Delta_G$  and  $\max(E + 6.25\Delta_E, E + 5\Delta_G)$ , respectively.

### Partial derivatives

Derivatives of the broadened cross section (or transmission)  $f_{GE}$  with respect to  $\Delta t_G$ ,  $\Delta L$ , or  $\Delta t_E$  are found exactly as in the previous two subsections, with  $f_G$  and  $f_{\exp}$  replaced by  $f_{GE}$ .