

### II.D.2. Derivatives for MLBW and SLBW Approximations

From the form of the cross sections in Eqs. (II B3 a.1), (II B3 a.5), and (II B3 a.6), we note that there are only four expressions in which the resonance energies or widths occur. These expressions are denoted as follows:

$$A_{1c\lambda} = \Gamma_{\lambda c} (E - E_\lambda) / d_\lambda , \quad (\text{II D2.1})$$

$$A_{2c\lambda} = \Gamma_{\lambda c} \bar{\Gamma}_{\lambda\gamma} / d_\lambda , \quad (\text{II D2.2})$$

$$A_{3cc'\lambda} = \Gamma_{\lambda c} \Gamma_{\lambda c'} / d_\lambda , \quad (\text{II D2.3})$$

and 
$$A_{4c\lambda} = \Gamma_{\lambda c} \Gamma_\lambda / d_\lambda , \quad (\text{II D2.4})$$

where  $d$  is given by Eq. (II B3 a.4) as

$$d_\lambda = (E - E_\lambda)^2 + (\Gamma_\lambda / 2)^2 . \quad (\text{II D2.5})$$

Equation (II D2.4) is actually redundant, since

$$A_{4c\lambda} = \sum_{c'} A_{3cc'\lambda} + A_{2c\lambda} . \quad (\text{II D2.6})$$

As discussed in Section II.D, the assumption in the SAMMY code is that the  $u$ -parameters are independent and the  $p$ -parameters are derived quantities. Thus we need only evaluate partial derivatives of  $A_{1c\lambda}$ ,  $A_{2c\lambda}$ , and  $A_{3cc'\lambda}$  with respect to the  $u$ -parameters (i.e., with respect to the partial-width amplitudes and to the square root of the resonance energy). These derivatives may be written as follows:

$$\frac{\partial A_{1c\lambda}}{\partial \sqrt{E_\lambda}} = 2\sqrt{E_\lambda} \Gamma_{\lambda c} \left\{ -1 + 2(E - E_\lambda)^2 / d_\lambda \right\} / d_\lambda , \quad (\text{II D2.7})$$

$$\frac{\partial A_{2c\lambda}}{\partial \sqrt{E_\lambda}} = 4\sqrt{E_\lambda} \Gamma_{\lambda c} \bar{\Gamma}_{\lambda\gamma} (E - E_\lambda) / d_\lambda^2 , \quad (\text{II D2.8})$$

$$\frac{\partial A_{3cc'\lambda}}{\partial \sqrt{E_\lambda}} = 4\sqrt{E_\lambda} \Gamma_{\lambda c} \Gamma_{\lambda c'} (E - E_\lambda) / d_\lambda^2 , \quad (\text{II D2.9})$$

$$\frac{\partial A_{1c\lambda}}{\partial \bar{\gamma}_{\lambda\gamma}} = -2\bar{\gamma}_{\lambda\gamma} \Gamma_{\lambda c} (E - E_\lambda) \Gamma_\lambda / d_\lambda^2 , \quad (\text{II D2.10})$$

$$\frac{\partial A_{2c\lambda}}{\partial \bar{\gamma}_{\lambda\gamma}} = 2\bar{\gamma}_{\lambda\gamma} \Gamma_{\lambda c} \left\{ 2 - \bar{\Gamma}_{\lambda\gamma} \Gamma_{\lambda} / d_{\lambda} \right\} / d_{\lambda} \quad , \quad (\text{II D2.11})$$

$$\frac{\partial A_{3cc'\lambda}}{\partial \bar{\gamma}_{\lambda\gamma}} = -2\bar{\gamma}_{\lambda\gamma} \Gamma_{\lambda c} \Gamma_{\lambda c'} \Gamma_{\lambda} / d_{\lambda}^2 \quad , \quad (\text{II D2.12})$$

$$\frac{\partial A_{1c\lambda}}{\partial \gamma_{\lambda c''}} = 2\Gamma_{\lambda c} (E - E_{\lambda}) \left\{ \delta_{cc''} - \Gamma_{\lambda} \Gamma_{\lambda c''} / (2d_{\lambda}) \right\} / \left\{ \gamma_{\lambda c''} d_{\lambda} \right\} \quad , \quad (\text{II D2.13})$$

$$\frac{\partial A_{2c\lambda}}{\partial \gamma_{\lambda c''}} = 2\Gamma_{\lambda c} \bar{\Gamma}_{\lambda\gamma} \left\{ \delta_{cc''} - \Gamma_{\lambda} \Gamma_{\lambda c''} / (2d_{\lambda}) \right\} / \left\{ \gamma_{\lambda c''} d_{\lambda} \right\} \quad , \quad (\text{II D2.14})$$

and

$$\frac{\partial A_{3cc''\lambda}}{\partial \gamma_{\lambda c''}} = 2\Gamma_{\lambda c} \Gamma_{\lambda c'} \left\{ \delta_{cc''} + \delta_{c'c''} - \Gamma_{\lambda} \Gamma_{\lambda c''} / (2d_{\lambda}) \right\} / \left\{ \gamma_{\lambda c''} d_{\lambda} \right\} \quad , \quad (\text{II D2.15})$$

All other derivatives are zero.