II. B. 1. a. Energy-differential cross sections

The observable cross sections are found in terms of X by first substituting Eqs. (II A.4, II A.5, and II B1.3) into Eq. (II A.1), summing over spin groups (i.e., over J^π), and then summing over all channels corresponding to those particle pairs and spin groups. If X^r represents the real part and X^i the imaginary part of X, then the angle-integrated (but energy-differential) cross section for the interaction that leads from particle pair α to particle pair α' has the form

$$\sigma_{\alpha,\alpha'}(E) = \frac{4\pi}{k_\alpha^2} \sum_J g_{J\alpha} \sum_c \left[\left(\sin^2\varphi_c \left(1-2X^i_{cc}\right) - X^r_{cc}\sin\left(2\varphi_c\right)\right)\delta_{\alpha,\alpha'} + \sum_{c'} \left\{ X^{i\,2}_{cc'} + X^{r\,2}_{cc'} \right\} \right] \quad . \tag{II B1 a.1}$$

(This formula is accurate only for cases in which one of particles in α is a neutron; however, both particles in α' may be charged.)

In Eq. (II B1 a.1) the summations are over those channels c and c' {of the spin group defined by J^π} for which the particle pairs are, respectively, α and α'. More than one "incident channel" $c = (\alpha, l, s, J)$ can contribute to this cross section, for example when both $l = 0$ and $l = 2$ are possible, or when, in the case of incident neutrons and non-zero spin target nuclei, both channel spins are allowed. Similarly, there may be several "exit channels" $c' = \left(\alpha', l', s', J'\right)$, depending on the particular reaction being calculated (e.g., elastic, inelastic, fission).

The total cross section (for non-Coulomb initial states) is the sum of Eq. (II B1 a.1) over all possible final-state particle-pairs α', assuming the scattering matrix is unitary (i.e., assuming that the sum over c' of $\left|U_{cc'}\right|^2 = 1$). Written in terms of the X matrix, the total cross section has the form

$$\sigma_{total}(E) = \frac{4\pi}{k_\alpha^2} \sum_J g_{J\alpha} \sum_c \left[\sin^2\varphi_c + X^i_{cc}\cos\,(2\varphi_c) - X^r_{cc}\sin\,(2\varphi_c) \right] \quad , \tag{II B1 a.2}$$

where again the sum over c includes only those channels of the J^π spin group for which the particle pair is α.

The angle integrated elastic cross section is given by

$$\sigma_{elastic}(E) = \frac{4\pi}{k_\alpha^2} \sum_J g_{J\alpha} \sum_c \left[\sin^2\varphi_c \left(1-2X^i_{cc}\right) - X^r_{cc}\sin\left(2\varphi_c\right) + \sum_{c'}\left\{ X^{i\,2}_{cc'} + X^{r\,2}_{cc'} \right\} \right] \quad . \tag{II B1 a.3}$$

In this case, both c and c' are limited to those channels of the J^π spin group for which the particle-pair is α; again, there may be more than one such channel for a given spin group.

Similarly, the reaction cross section from particle pair α to particle pair α' (where α' is not equal to α) is

$$\sigma_{reaction}(E) = \frac{4\pi}{k_\alpha^2} \sum_J g_{J\alpha} \sum_c \sum_{c'} \left[X^{i\ 2}_{cc'} + X^{r\ 2}_{cc'} \right] \ . \qquad \text{(II B1 a.4)}$$

Here c is restricted to those channels of the J^π spin group from which the particle pair is α, and c' to those channels for which the particle-pair is α'.

The absorption cross section has the form

$$\sigma_{absorption}(E) = \frac{4\pi}{k_\alpha^2} \sum_J g_{J\alpha} \sum_c \left[X^i_{cc} - \sum_{c'} \left\{ X^{i\ 2}_{cc'} + X^{r\ 2}_{cc'} \right\} \right] \ . \qquad \text{(II B1 a.5)}$$

Here both the sum over c and the sum over c' include <u>all</u> <u>incident</u> particle channels (i.e., particle pair α only) for the J^π spin group.

The capture cross section for the eliminated radiation channels can be calculated directly as

$$\sigma_{capture}(E) = \frac{4\pi}{k_\alpha^2} \sum_J g_{J\alpha} \sum_{inc\,c} \left[X^i_{cc} - \sum_{all\,c'} \left\{ X^{i\ 2}_{cc'} + X^{r\ 2}_{cc'} \right\} \right] \ , \qquad \text{(II B1 a.6)}$$

or may be found by subtracting the sum of all reaction cross sections from the absorption cross section. In Eq. (II B1 a.6), the sum over c includes all incident particle channels for the J^π spin group, and the sum over c' includes <u>all</u> particle channels, both incident and exit, for that spin group.