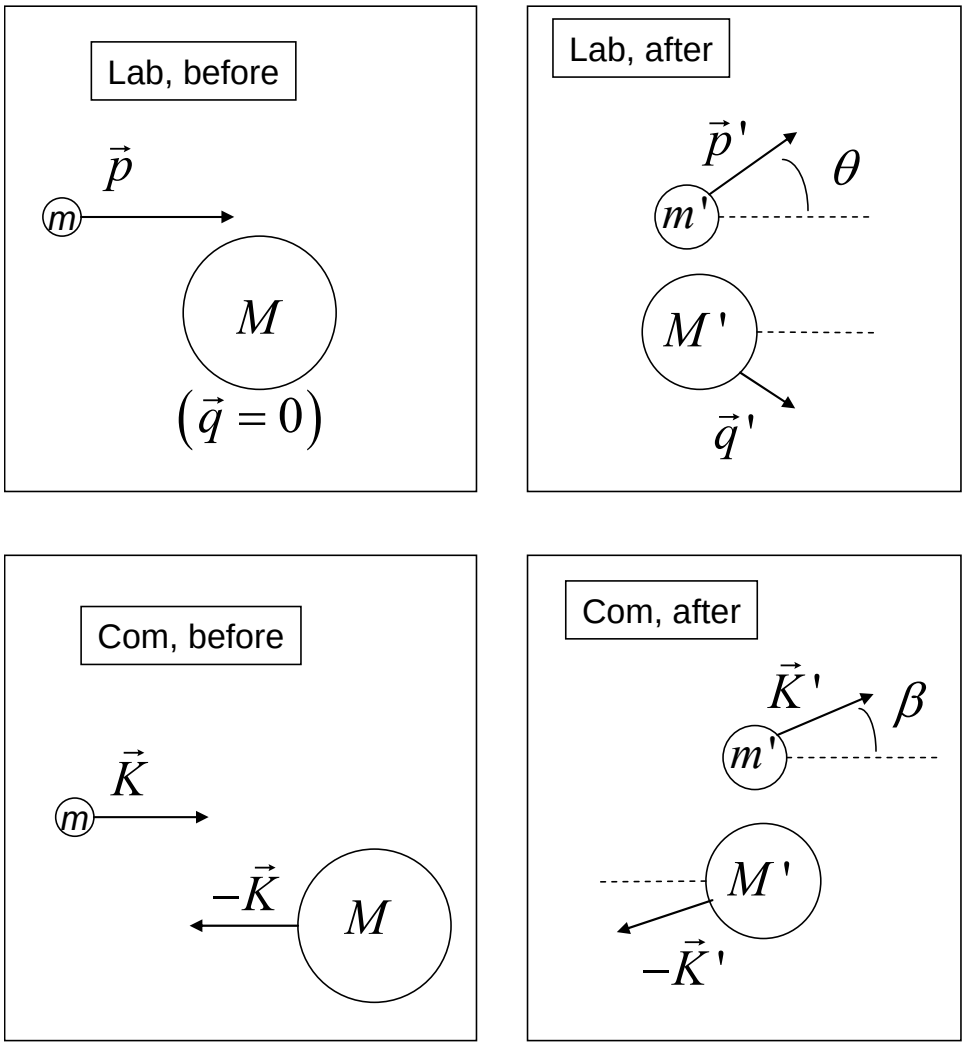
II.C.2. Kinematics

The center-of-mass (COM) momenta, $K = \hbar k$ (initial) and $K' = \hbar k'$ (after the interaction), needed in the formulae for cross sections (see Section II.A) may be found in terms of laboratory energies E and E' by utilizing conservation of energy and momentum. The interaction is shown schematically in Figure II C 2.1, in both the laboratory (lab) and COM systems.

In this section we summarize the relevant equations relating the various momenta, energies, and angles involved in the description of the reaction. Details of the derivations are given in Section II.C.2.a.

Figure II C 2.1. Schematic of particle pairs for kinematics calculation.



Nomenclature: The two particles have mass *m* and *M* before the interaction (i.e., in the incident channel); the exit channel may contain completely different particles. The initial momentum of the incident particle is $\vec{p}$; the target particle is stationary (momentum $\vec{q} = 0$); in the COM, the initial momentum of the incident particle is $\vec{K} = \hbar\vec{k}$, and the target particle's momentum is $-\vec{K}$. Primes refer to values after the interaction (in the exit channel). Quantities measured during an experiment are incident laboratory energy $E = p^2/2m$, laboratory energy of the exiting particle $E' = p'^2/2m'$, and laboratory angle θ relative to the incident direction. All other quantities will be specified in terms of these quantities.

Equation numbers in the rest of this section relate to the derivation in the following section.

The Q-value, Eq. (II C2 a.6), is defined as

$$Q \equiv m + M - m' - M' \tag{II C2.1}$$

and is related to the laboratory threshold energy , Eq. (II C2 a.24), here denoted by Ξ, via

$$\Xi = -\frac{m+M}{M}Q \quad . \tag{II C2.2}$$

The initial momentum *K* in the COM, Eq. (II C2 a.2), is given by

$$K = \hbar k = \frac{M}{m+M}\sqrt{2mE} \quad , \tag{II C2.3}$$

and the final COM momentum K', Eq. (II C2 a.8), by

$$K' = \sqrt{\frac{2m'M'}{(m'+M')}\frac{M}{(m+M)}\left[E-\Xi\right]} \quad . \tag{II C2.4}$$

The laboratory energy of the outgoing particle, Eq. (II C2 a.25), is equal to

$$E' = \frac{M'}{(m'+M')}\frac{M}{(m+M)}\left\{\gamma\mu + \sqrt{1-\gamma^2\left(1-\mu^2\right)}\right\}^2\left[E-\Xi\right] \quad , \tag{II C2.5}$$

in which $\mu = \cos\theta$, and γ is given in Eq. (II C2 a.26) as

$$\gamma^2 = \frac{m'}{M'}\frac{m}{M}\frac{m'+M'}{m+M}\frac{E}{E-\Xi} \quad . \tag{II C2.6}$$

The relationships between the COM and lab angles are, from Eqs. (II C2 a.29) and (II C2 a.31)

$$\nu = -\gamma\left(1-\mu^2\right)+\mu\sqrt{1-\gamma^2\left(1-\mu^2\right)} \qquad \text{(II C2.7)}$$

and

$$\mu = \frac{\nu+\gamma}{\sqrt{1+\gamma^2+2\gamma\nu}} \quad , \qquad \text{(II C2.8)}$$

where $\nu = \cos\beta$ and β is the COM angle. The transformation of angle-differential cross sections from COM to lab involves the derivative of ν with respect to μ, as given by Eq. (II C2 a.33):

$$\frac{d\nu}{d\mu} = \frac{\left(\mu\gamma+\sqrt{1-\gamma^2\left(1-\mu^2\right)}\right)^2}{\sqrt{1-\gamma^2\left(1-\mu^2\right)}} \quad . \qquad \text{(II C2.9)}$$

See Section II.B.1.b for details on the calculations of angular distributions with the Reich-Moore formulation of R-matrix theory.

In older versions of input to the SAMMY code (when particle-pair input is not used), the excitation energy can be specified either in the laboratory system (as in Eq. (II C2.2)) or in the center-of-mass system (as $-Q$); SAMMY will make the appropriate conversions. The default is laboratory. Users who wish to override the default (or who wish to keep a reminder handy) should include (in INPut file) the phrase

```
CM NON-COULOMB EXCITation energies, or
LAB NON-COULOMB EXCItation energies
```

as needed for the non-Coulomb case and

```
CM COULOMB EXCITATIOn energies, or
LAB COULOMB EXCITATIon energies
```

for use with charged-particle channels.

When using the key-word particle-pair input option (card set 4 of Table VI A.1), it is possible to specify the Q-value (equivalent to the negative of the center-of-mass excitation energy) rather than the excitation energy. It is recommended that Q-value rather than excitation energy be given, to avoid any ambiguity when more than one nuclide is present in the target.

Within SAMMY, the conversion factors from laboratory energy to COM momenta are calculated in subroutine Fixrad in segment/subdirectory "old" (and also used in segment "new") and stored in an array "Zke" which must then be multiplied by the square root of the energy (minus the adjusted Q value) to give k or k'. Appropriate numerical constants are included to facilitate conversion from units of eV (for energy) to inverse Fermi (for wave number, which is momentum divided by $\hbar$). Values for constants are described in Section IX.A of this report.