II.B.1.b. Angular distributions

Angular distributions (elastic, inelastic, or other reaction) cross sections for incident neutrons can be calculated from Reich-Moore resonance parameters. Following Blatt and Biedenharn [JB52] with some notational changes, the angular distribution cross section in the center-of-mass system may be written

$$\frac{d\,\sigma_{\alpha\alpha'}}{d\,\Omega_{CM}} = \sum_L B_{L\alpha'}(E)\,P_L(\cos\beta) \quad , \qquad \text{(II B1 b.1)}$$

in which the subscript $\alpha\alpha'$ indicates which type of cross section is being considered (i.e., α represents the entrance particle pair and α' represents the exit pair). P_L is the Legendre polynomial of degree L, and β is the angle of the outgoing neutron (or other particle) relative to the incoming neutron in the center-of-mass system. The coefficients $B_{L\alpha'}(E)$ are given by

$$\begin{aligned} B_{L\alpha'}(E) = \frac{1}{4k_\alpha^2} \sum_{J_1} \sum_{J_2} \sum_{l_1 s_1} \sum_{l'_1 s'_1} \sum_{l_2 s_2} \sum_{l'_2 s'_2} \frac{1}{(2i+1)(2I+1)} \\ \times\; G_{\{l_1 s_1 l'_1 s'_1 J_1\}\{l_2 s_2 l'_2 s'_2 J_2\}\,L} \quad \mathrm{Re}\left[(\delta_{c_1 c'_1} - U_{c_1 c'_1})\,(\delta_{c_2 c'_2} - U^*_{c_2 c'_2})\right] \quad , \end{aligned} \qquad \text{(II B1 b.2)}$$

in which the various summations are to be interpreted as follows:

(1) sum over all spin groups defined by spin J_1 and the implicit associated parity

(2) sum over all spin groups defined by spin J_2 and the implicit associated parity

(3) sum over the entrance channels c_1 belonging to the J_1 spin group and having particle pair α, with orbital angular momentum l_1 and channel spin s_1 [i.e., $c_1 = (\alpha, l_1, s_1, J_1)$]

(4) sum over the exit channels c'_1 in J_1 spin group with particle-pair α', orbital angular momentum l'_1, and channel spin s'_1 [i.e., $c'_1 = (\alpha', l'_1, s'_1, J_1)$]

(5) sum over entrance channels c_2 in J_2 spin group where $c_2 = (\alpha, l_2, s_2, J_2)$

(6) sum over exit channels c'_2 in J_2 spin group where $c'_2 = (\alpha', l'_2, s'_2, J_2)$

Also note that i and I are the spins of the two particles (projectile and target nucleus) in particle-pair α.

The geometric factor G can be exactly evaluated as a product of terms

$$G_{\{l_1 s_1 l'_1 s'_1 J_1\}\{l_2 s_2 l'_2 s'_2 J_2\}L} = A_{l_1 s_1 l'_1 s'_1; J_1}\; A_{l_2 s_2 l'_2 s'_2; J_2}\; D_{l_1 s_1 l'_1 s'_1 l_2 s_2 l'_2 s'_2;\, L J_1 J_2} \quad , \qquad \text{(II B1 b.3)}$$

where the factor $A_{l_1 s_1 l'_1 s'_1; J_1}$ is of the form

$$A_{l_1 s_1 l'_1 s'_1; J_1} = \sqrt{(2l_1+1)\;(2l'_1+1)}\;(2J_1+1)\;\Delta(l_1 J_1 s_1)\;\Delta(l'_1 J_1 s'_1) \quad . \qquad \text{(II B1 b.4)}$$

The expression for D is

$$D_{l_1 s_1 l'_1 s'_1 l_2 s_2 l'_2 s'_2 ; L J_1 J_2} = (2L+1)\ \Delta^2(J_1 J_2 L)\ \Delta^2(l_1 l_2 L)\ \Delta^2(l'_1 l'_2 L)$$
$$\times\, w(l_1 J_1 l_2 J_2, s_1 L)\ w(l'_1 J_1 l'_2 J_2, s'_1 L)\ \delta_{s_1 s_2}\ \delta_{s'_1 s'_2}\ (-1)^{s_1 - s'_1} \qquad \text{(II B1 b.5)}$$
$$\times\, \frac{n!\ \ (-1)^n}{(n-l_1)!\ (n-l_2)!\ (n-L)!}\ \ \frac{n'!\ \ (-1)^{n'}}{(n'-l'_1)!\ (n'-l'_2)!\ (n'-L)!}\quad ,$$

in which n is defined by

$$2n = l_1 + l_2 + L \quad ; \qquad \text{(II B1 b.6)}$$

D is zero if l_1+l_2+L is an odd number. A similar expression defines n'. The Δ^2 term is given by

$$\Delta^2(abc) = \frac{(a+b-c)!\ (a-b+c)!\ (-a+b+c)!}{(a+b+c+1)!}\quad , \qquad \text{(II B1 b.7)}$$

for which the arguments a, b, and c are to be replaced by the appropriate values given in Eqs. (II B1 b.4) and (II B1 b.5). The expression for $\Delta^2(abc)$ implicitly includes a selection rule for the arguments; that is, the quantized vector sum must hold,

$$\vec{a} + \vec{b} = \vec{c} \qquad \text{or} \qquad |a-b| \le c \le a+b \qquad \text{(II B1 b.8)}$$

with c being either integer or half-integer. The quantity w in Eq. (II B1 b.5) is defined as

$$w(l_1 J_1 l_2 J_2,\ s L) = \sum_{k=kmin}^{kmax} \frac{(-1)^{k+l_1+J_1+l_2+J_2}\ (k+1)!}{\big(k-(l_1+J_1+s)\big)!\ \big(k-(l_2+J_2+s)\big)!}$$
$$\times\, \frac{1}{\big(k-(l_1+l_2+L)\big)!\ \big(k-(J_1+J_2+L)\big)!} \qquad \text{(II B1 b.9)}$$
$$\times\, \frac{1}{(l_1+J_1+l_2+J_2-k)!\ (l_1+J_2+s+L-k)!\ (l_2+J_1+s+L-k)!}$$

(and similarly for the primed expression), where *kmin* and *kmax* are chosen such that none of the arguments of the factorials are negative. That is,

$$kmin = \max\left\{ (l_1+J_1+s),\ (l_2+J_2+s),\ (l_1+l_2+L),\ (J_1+J_2+L) \right\}$$
$$kmax = \min\left\{ (l_1+J_1+l_2+J_2),\ (l_1+J_2+s+L),\ (l_2+J_1+s+L) \right\}\quad . \qquad \text{(II B1 b.10)}$$

Single-channel case

For some situations, these equations can be greatly simplified. When the target spin is zero and there are no possible reactions (no fission, no inelastic, no other reactions), then each spin group will consist of a single channel (the elastic channel). In this case, the coefficients $B_{L\alpha\alpha'}(E)$ reduce to

$$B_{L\alpha\alpha}(E) = \frac{1}{4k_\alpha^2} \sum_{J_1} \sum_{J_2} \sum_{c_1=(\alpha\, l_1 s_1 J_1)} \sum_{c_2=(\alpha\, l_2 s_2 J_2)} G_{\{l_1 s_1 l_1 s_1 J_1\}\,\{l_2 s_2 l_2 s_2 J_2\}\,L} \times \frac{1}{(2i_a+1)\,(2i_b+1)} \operatorname{Re}\left[\,(1-U_{c_1 c_1})\;(1-U^*_{c_2 c_2})\,\right] \;, \tag{II B1 b.11}$$

where the existence of only one channel requires that the primed quantities of Eq.(II B1 b.2) be equal to the unprimed (e.g., $\alpha = \alpha'$). The geometric factor *G* becomes

$$G_{\{l_1 s_1 l_1 s_1 J_1\}\,\{l_2 s_2 l_2 s_2 J_2\}\,L} = A_{l_1 s_1 l_1 s_1 ; J_1}\; A_{l_2 s_2 l_2 s_2 ; J_2}\; D_{l_1 s_1 l_1 s_1 l_2 s_2 l_2 s_2 ;\, L\, J_1\, J_2} \;, \tag{II B1 b.12}$$

in which the factor *A* reduces to the simple form

$$A_{l_1 s_1 l_1 s_1 ; J_1} = (2l_1+1)\;(2J_1+1)\;\Delta^2(l_1\, J_1\, s_1) \;, \tag{II B1 b.13}$$

and the expression for *D* reduces to

$$D_{l_1 s_1 l_1 s_1 l_2 s_2 l_2 s_2 ;\, L\, J_1\, J_2} = (2L+1)\;\Delta^2(J_1\, J_2\, L)\;\Delta^4(l_1\, l_2\, L) \times w^2(l_1\, J_1\, l_2\, J_2, s_1\, L)\;\delta_{s_1 s_2} \left[\frac{n!}{(n-l_1)!\;(n-l_2)!\;(n-L)!}\right]^2 \;, \tag{II B1 b.14}$$

in which *n* is again defined as in Eq. (II B1 b.6).