III.C.3.a. Components of the RPI / GELINA / nTOF resolution function

1. Electron burst

The electron burst from the RPI linac may be described by a Gaussian function in time, of the form:

$$I_1\left(t\right)=\frac{w}{\sqrt{\pi}}e^{-w^2t^2} \quad , \qquad \text{(III C3 a.1)}$$

where $2\sqrt{\ln 2}\,/\,w=p$ is the full width at half max of the burst. Normalization is unity for this function.

2. Target plus detector

The RPI transmission resolution function, which represents the combined components for the "bounce target" and transmission detector, has been found by RPI researchers [BM96] to be best described by the sum of a chi-squared function with six degrees of freedom plus two exponential terms. A similar function (with different values for the parameters) describes the bounce target plus capture detector.

The original RPI function had constant values for A_3 and A_5. The GELINA and nTOF resolution functions proposed by Gunsing [FG05] use energy dependent values of the A_3 and A_5 parameters.

Specifically, the RPI resolution function has the form

$$\begin{aligned} I_2\left(t\right)=A_0\Bigg\{&\frac{\left(t+\tau\right)^2}{2!\Lambda^3}e^{-(t+\tau)/\Lambda}+A_1\left[A_2e^{-A_3(t+t_0)}+A_4e^{-A_5(t+t_0)}\right]X\left(t\right)\\ &+\sum_{i=1}^{5}B_{2i-1}e^{-B_{2i}(t+t_0)}\Bigg\} \quad , \end{aligned} \qquad \text{(III C3 a.2)}$$

in which the function $X\left(t\right)$ is zero if the quantity within the square brackets (the sum of the exponential terms) is negative, and unity otherwise. Likewise the χ^2 function is assumed to have zero value when the exponent is positive (i.e., when $t+\tau<0$). The value of A_0 is chosen to give an overall normalization of unity for this function. Parameters Λ, τ, A_1, A_3, and A_5 are functions of energy, the specific forms being, respectively,

$$\Lambda\left(E\right)=\Lambda_0+\Lambda_1\ln\left(E\right)+\Lambda_2\left[\ln\left(E\right)\right]^2+\Lambda_3E^{\Lambda_4} \quad , \qquad \text{(III C3 a.3)}$$

$$\tau(E) = \tau_1 e^{-\tau_2 E} + \tau_3 e^{-\tau_4 E} + \tau_5 + \tau_6 E^{\tau_7} \quad , \qquad \text{(III C3 a.4)}$$

and

$$A_i(E) = \left\{ a_{i1} e^{-a_{i2} E} + a_{i2} e^{-a_{i4} E} + a_{i5} + a_{i6} E^{a_{i7}} \right\} \alpha_i \quad , \qquad \text{(III C3 a.5)}$$

where *i* represents 1, 3, or 5, and α_i is 1 for $i = 1$ but may be either unity or $\sqrt{E}$ for $i = 3$ or 5. All other quantities in Eq.(III C3 a.2) are independent of energy.

As many as five exponential terms may be included in the sum over *i*; it is implicitly assumed that the coefficients of the exponentials (B_{2i-1}) and the coefficients of time within the exponentials (B_{2i}) are positive numbers. These terms were added in an early attempt to provide a useful form for the GELINA and nTOF resolution function; they have been retained in order to permit additional flexibility for the analyst.

3. Time-of-flight channel width

The time-of-flight channel width may be modeled as a rectangular distribution of width *c*. The time distribution due to the finite channel width is therefore assumed to be

$$I_3(t) = \begin{cases} 1/c & \text{for } -c/2 < t < c/2 \\ 0 & \text{otherwise} \end{cases} \quad . \qquad \text{(III C3 a.6)}$$

The channel width *c* may be energy dependent. For constant values of *c* within an energy range, the input is described in Table VI.B, card set 14, line 20. This is appropriate, for example, for data having “crunch boundaries.” (In Europe, the French word “accordeon” is often used to denote the system of crunch boundaries.)

When the channel width varies continuously with energy, for example, for data from the nTOF facility, then the channel width (or bin width) is expressed as “n bins per decade.” That is, in an energy decade from 10^k to $10^{(k+1)}$ for integer *k*, there are *n* bins equally spaced on a logarithmic scale. The energy limits for the i^{th} bin in this decade are given by

$$E_{k,i} = 10^{k+\frac{i-1}{n}} \le E = 10^{k+\frac{i-1+\varepsilon}{n}} \le E_{k,i+1} = 10^{k+\frac{i}{n}} \quad , \qquad \text{(III C3 a.7)}$$

where $i = 1$ to *n*, and ε is a positive number between 0 and 1. Converting to time limits, using $t = \tau / \sqrt{E}$ where τ is a constant whose value is unimportant for this discussion, we find

$$t_{k,i+1} = \frac{\tau}{\sqrt{10^{k+\frac{i}{n}}}} \le t = \frac{\tau}{\sqrt{10^{k+\frac{i-1+\varepsilon}{n}}}} \le t_{k,i} = \frac{\tau}{\sqrt{10^{k+\frac{i-1}{n}}}} \quad , \qquad \text{(III C3 a.8)}$$

so that the channel width *c* may be found from

$$c = t_{k,i} - t_{k,i+1} = t\left\{ \frac{1}{\sqrt{10^{\frac{\varepsilon}{n}}}} - \frac{1}{\sqrt{10^{\frac{1-\varepsilon}{n}}}} \right\} = t\left\{ 10^{-\frac{\varepsilon}{2n}} - 10^{\frac{-1+\varepsilon}{2n}} \right\} \quad . \qquad \text{(III C3 a.9)}$$

To calculate this quantity in SAMMY, we consider the value at $\varepsilon = 0$,

$$c_0 \;\cong\; t\;\left\{1 - 10^{-1/(2n)}\right\} \quad , \qquad \text{(III C3 a.10)}$$

and at $\varepsilon = 1$,

$$c_1 \;\cong\; t\;\left\{10^{\,1/(2n)} - 1\right\} \quad . \qquad \text{(III C3 a.11)}$$

In either case, for large *n* these expressions may be expanded to first order in 1/*n* to give

$$c_0 \;=\; t\;\left\{1 - 10^{-1/(2n)}\right\} \;\cong t\left\{1 - \left[1 - \left(\frac{1}{2n}\ln\left(10\right)\right)\right]\right\} = \frac{t}{2n}\ln\left(10\right) \qquad \text{(III C3 a.12)}$$

and

$$c_1 \;=\; t\;\left\{10^{\,1/(2n)} - 1\right\} \;\cong t\left\{\,1\, + \,\left(\frac{1}{2n}\ln\left(10\right)\right) - 1\right\} = \frac{t}{2n}\ln\left(10\right) \quad . \qquad \text{(III C3 a.13)}$$

Because these two values agree to first order in 1/*n*, and *n* is large (~5000), it is therefore sufficient to use the approximation

$$c \cong t\ln\left(10\right)/\left(2n\right) \qquad \text{(III C3 a.14)}$$

rather than to spend the not-insignificant amount of computer time to generate exact values of *k* and *i* (and therefore of *c*) for each value of *E*.

Input for the continuously varying definition of channel width *c* is given in Table VI.B, card set 14, line 19.