II. SCATTERING THEORY

Details of scattering theory have been well understood since the middle of the previous century, when they were summarized in a review article by Lane and Thomas [AL58]. A wealth of additional reference material is available to the student of scattering theory; only a few are listed here. The text by Foderaro [AF71] provides a more elementary introduction to the subject. One publication by Fröhner [FF80] is based on lectures presented at the International Centre for Theoretical Physics (ICTP) Winter Courses on Nuclear Physics and Reactors, 1978; this is a comprehensive and useful guide to applied neutron resonance theory. It includes a variety of topics, including preparation of data, various approximations to scattering theory, Doppler broadening, experimental complications, data-fitting procedures, and statistical tests. Another Fröhner paper [FF00] is somewhat more theoretical, and covers many aspects of data fitting in the resonance region.

The particular aspect of scattering theory with which we are concerned is the R-matrix formalism. A summary of the underlying principles is given here.

R-matrix theory is a mathematically rigorous phenomenological description of what is actually seen in an experiment (i.e., the measured cross section). The theory is not a model of neutron-nucleus interaction, in the sense that it makes no assumptions about the underlying physics of the interaction. Instead it parameterizes the measurement in terms of quantities such as the interaction radii and boundary conditions, resonance energies and widths, and quantum numbers; values for these parameters may be determined by fitting theoretical calculations to observed data. The theory is mathematically correct, in that it is analytic, unitary, and rigorous; nevertheless, in practical applications, the theory is always approximated in some fashion.

R-matrix theory is based on the following assumptions: (1) the applicability of non-relativistic quantum mechanics; (2) the absence or unimportance of all processes in which more than two product nuclei are formed; (3) the absence or unimportance of all processes of creation or destruction; and (4) the existence of a finite radial separation beyond which no nuclear interactions occur, although Coulomb interactions are given special treatment. [In practical applications two of these four assumptions may be violated in one degree or another: (1) The theory may be used for relativistic neutron energies, and corrected for relativistic effects; nevertheless, non-relativistic quantum mechanics is assumed. (2) A fission experiment with more than two final products is treated as a two-step process. That is, the immediate result of the neutron-nuclide interaction is assumed to be limited to two final products, at least one of which decays prior to detection.]

R-matrix theory is expressed in terms of channels, where a channel is defined as a pair of (incoming or outgoing) particles, plus specific information relevant to the interaction between the two particles. A schematic depicting entrance and exit channels is shown in Figure II.1. Note that entrance channels can also occur as exit channels, but some exit channels (e.g., fission channels) do not occur as entrance channels. Two interacting particles are shown in the portion of the figure that is labeled “Interior Region”; here the particles are separated by less than the interaction radius *a*.

In Section II.A, general equations of scattering theory are presented and their derivations discussed. The fundamental R-matrix equations are presented. Section II.A.1 gives a detailed derivation of the equations for a simple case. Section II.A.2 shows the relationship between the R-matrix and the A-matrix, which is another common representation of scattering theory.

The approximations to R-matrix theory available in the SAMMY code are detailed in Section II.B. The recommended choice for most applications is the Reich-Moore approximation, described in Section II.B.1. For some applications, the Reich-Moore approximation is inadequate; for those cases, a method for using SAMMY's Reich-Moore approximation to mimic the full (exact) R-matrix is presented Section II.B.2. Two historically useful but now obsolete approximations are single-level and multilevel Breit Wigner (SLBW and MLBW), discussed in Section II.B.3. Provisions for including non-compound (direct) effects are discussed in Section II.B.4.

In Section II.C, details are given for the SAMMY nomenclature and other conventions, for transformations to the center-of-momentum system, and for the calculation of penetrability, shift factors, and hard-sphere phase shifts in both Coulomb and non-Coulomb cases.

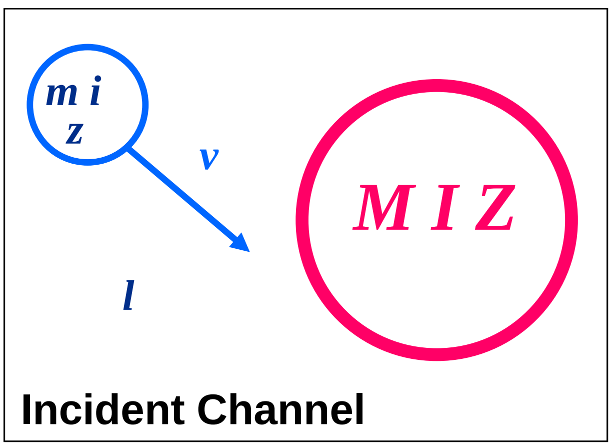

Figure II.1. Schematic of entrance and exit channels as used in scattering theory. For the interior region (with separation distance $r < a$), no assumptions are made about the nature of the interaction. In the figure, *m*, *i*, and *z* refer to the mass, spin, and charge of the incident particle while *M*, *I* and *Z* refer to the target particle. Orbital angular momentum is denoted by *l* and velocity by *v*. Primes are used for post-collision quantities.