III.E.1.a. Transmission through a non-uniform sample

Occasionally the sample is known to be non-uniform in thickness, generally thicker in the center and thinner on the edges. To model this situation, we assume that the sample is a circular disk of average thickness n (atoms/barn) or $\bar{Z}$ (cm), with $n = N\,\bar{Z}$. Actual thickness is specified by the user in piece-wise fashion as a function of distance R from the center, as

$$Z\left(R_i\right) \;=\; Z_i \quad \text{for} \quad i=1 \text{ to } M, \quad \text{with } R_{i=1}=0 \quad , \tag{III E1 a.1}$$

where the units on the R_i and Z_i are unimportant since SAMMY will rescale these. At arbitrary values of R, the thickness is given by linear interpolation between the specified points,

$$Z\left(R\right) \;=\; \frac{R-R_i}{R_{i+1}-R_i} Z_{i+1} + \frac{R_{i+1}-R}{R_{i+1}-R_i} Z_i \;=\; a_i + b_i R \quad , \tag{III E1 a.2}$$

where

$$a_i = \frac{R_{i+1}Z_i - R_i Z_{i+1}}{R_{i+1}-R_i} \qquad \text{and} \qquad b_i = \frac{Z_{i+1}-Z_i}{R_{i+1}-R_i} \quad . \tag{III E1 a.3}$$

Under these assumptions, the measured transmission may be modeled as the average over the area of the sample, as

$$\begin{aligned} T \;&=\; \frac{1}{\pi R_M^2} \int_0^{R_M} R\,dR \int_0^{2\pi} d\varphi\, e^{-N\sigma Z(R)} \\ &=\; \frac{2}{R_M^2} \sum_{i=2}^{M} \frac{1}{\left(N\sigma b_i\right)^2} \left[e^{-N\sigma Z_{i-1}} \left(N\sigma b_i R_{i-1}+1\right) - e^{-N\sigma Z_i} \left(N\sigma b_i R_i + 1\right) \right] \quad . \end{aligned} \tag{III E1 a.4}$$

When b_i is small, the result in Eq. (III E1 a.4) must be modified to avoid singularities. Rewriting this equation slightly gives a convenient form for small b_i ,

$$T \;=\; \frac{2}{R_M^2} \sum_{i=2}^{M} \frac{e^{-N\sigma Z_i}}{\left(N\sigma b_i\right)^2} \left[e^{N\sigma b_i\left(R_i - R_{i-1}\right)} \left(N\sigma b_i R_{i-1}+1\right) - \left(N\sigma b_i R_i + 1\right) \right] \quad . \tag{III E1 a.5}$$

For small arguments, an exponential can be written in the form

$$e^x = 1 + Ax \qquad A = 1 + Bx \qquad B = \tfrac{1}{2} + Cx \qquad C = \tfrac{1}{6} + Dx \quad . \tag{III E1 a.6}$$

Using this form for the second exponential puts Eq. (III E1 a.5) into the form

$$T \;=\; \frac{2}{R_M^2} \sum_{i=2}^{M} e^{-N\sigma Z_i} \left(R_i - R_{i-1}\right) \left[R_{i-1}A + \left(R_i - R_{i-1}\right) B \right] \quad . \tag{III E1 a.7}$$

As one test to see whether this form is correct, consider the case for which all Z_i are exactly equal. For this situation, $A \to 1$ and $B \to \frac{1}{2}$, and the transmission becomes

$$T \;\to\; \frac{2}{R_M^2} e^{-N\sigma\bar{Z}} \sum_{i=2}^{M} \left(R_i - R_{i-1}\right) \left[R_{i-1} + \left(R_i - R_{i-1}\right)\tfrac{1}{2} \right] \;=\; e^{-n\sigma} \quad . \tag{III E1 a.8}$$

This is identical to the uniform-sample case. QED.

Input for this option is accomplished in the same manner as for the multiple-scattering correction in the case of non-uniform sample (not yet implemented in the code). The user provides values for R_i and Z_i of Eq. (III E1 a.1). It is not necessary for the Z_i to be normalized so that the average is $\bar{Z}$, only that the ratios be correct; SAMMY will take care of normalization. Details for input of the non-uniform thickness are given in card set 11 of the PARameter file.

The derivative of the transmission with respect to the total cross section σ is also needed. From Eq. (III E1 a.4), it is clear that this derivative is

$$\begin{aligned}\frac{dT}{d\sigma} = \frac{2}{R_M^2} \sum_{i=2}^{M} \Bigg\{ & \frac{-2/\sigma}{\left(N\sigma b_i\right)^2}\left[e^{-N\sigma Z_{i-1}}\left(N\sigma b_i R_{i-1}+1\right)-e^{-N\sigma Z_i}\left(N\sigma b_i R_i+1\right)\right] \\ & +\frac{1}{\left(N\sigma b_i\right)^2}\left[-NZ_{i-1}e^{-N\sigma Z_{i-1}}\left(N\sigma b_i R_{i-1}+1\right)+NZ_i e^{-N\sigma Z_i}\left(N\sigma b_i R_i+1\right)\right] \\ & +\frac{1}{\left(N\sigma b_i\right)^2}\left[e^{-N\sigma Z_{i-1}}\left(Nb_i R_{i-1}\right)-e^{-N\sigma Z_i}\left(Nb_i R_i\right)\right]\Bigg\} \qquad \text{(III E1 a.9)} \\ = \frac{1}{\sigma}\frac{2}{R_M^2} \sum_{i=2}^{M}\left(N\sigma b_i\right)^{-2} \Big\{ & -e^{-N\sigma Z_{i-1}}\left[\left(1+N\sigma Z_{i-1}\right)N\sigma b_i R_{i-1}+\left(2+N\sigma Z_{i-1}\right)\right] \\ & +e^{-N\sigma Z_i}\left[\left(1+N\sigma Z_i\right)N\sigma b_i R_i+\left(2+N\sigma Z_i\right)\right]\Big\} \quad .\end{aligned}$$

The small-b expansion for this expression is

$$\begin{aligned}\frac{dT}{d\sigma} = \frac{1}{\sigma}\frac{2}{R_M^2} \sum_{i=2}^{M}\left(N\sigma b_i\right)^{-2} e^{-N\sigma Z_i}\Big\{ & -e^{N\sigma b_i\left(R_i-R_{i-1}\right)}\left[\left(1+N\sigma Z_{i-1}\right)N\sigma b_i R_{i-1}+\left(2+N\sigma Z_{i-1}\right)\right] \\ & +\left[\left(1+N\sigma Z_i\right)N\sigma b_i R_i+\left(2+N\sigma Z_i\right)\right]\Big\} \quad ,\end{aligned}$$

which becomes

$$\begin{aligned}\frac{dT}{d\sigma} = \frac{2}{\sigma R_0^2} \sum_{i=2}^{M} e^{-N\sigma Z_i}\Big\{ & R_i^2-R_{i-1}^2-\left(R_i-R_{i-1}\right)\Big[A\left(2R_i+N\sigma Z_{i-1}R_{i-1}\right) \\ & +B\left(2+N\sigma a_i\right)\left(R_i-R_{i-1}\right)\Big]\Big\} \quad . \qquad \text{(III E1 a.10)}\end{aligned}$$

In the limit as all the b_i become zero, this is

$$\begin{aligned}\frac{dT}{d\sigma} & \to \frac{2}{\sigma R_M^2} \sum_{i=2}^{M} e^{-n\sigma}\left\{R_i^2-R_{i-1}^2-\left(R_i-R_{i-1}\right)\left[\left(2R_{i-1}+n\sigma R_{i-1}\right)+\tfrac{1}{2}\left(2+n\sigma\right)\left(R_i-R_{i-1}\right)\right]\right\} \\ & \to \frac{-n}{R_M^2}\, e^{-n\sigma} \sum_{i=2}^{M}\left\{\left(R_i^2-R_{i-1}^2\right)\right\} = -n\, e^{-n\sigma} \quad .\end{aligned}$$

(III E1 a.11)

Again, this is identical to the uniform-sample case. QED.

Test case tr178 has examples of this feature.