## II.D.1. Derivatives for Reich-Moore Approximation

The derivative of the cross section with respect to a resonance parameter is found by making use of the chain rule:

$$\frac{\partial \sigma}{\partial u_i} = \sum_{\substack{\mu \le \nu \\ \omega \le \tau}} \frac{\partial R_{\mu\nu}}{\partial u_i} \frac{\partial W_{\omega\tau}}{\partial R_{\mu\nu}} \frac{\partial U_{\omega\tau}}{\partial W_{\omega\tau}} \frac{\partial \sigma}{\partial U_{\omega\tau}} \quad , \tag{II D1.1}$$

where the index $J$ has been suppressed, since it is fixed for a given parameter $u$. Each term in this expression is evaluated separately.

Derivatives of cross sections with respect to the scattering matrix $U$, derivatives of $U$ with respect to $W$, and derivatives of $W$ with respect to $R$ are found in Section II.D.1.a. Derivatives of $R$ with respect to resonance parameters are given in Section II.D.1.b.

Derivatives of the cross sections with respect to the channel radius require additional terms beyond those in Eq. (II D1.1), because the radius is also used to determine the hard-sphere phase shift. These derivatives are discussed in Section II.D.1.c.

Derivatives of $R$ with respect to the variables of the logarithmic parameterization of the external R-function (defined in Section II.B.1.d) are given in Section II.D.1.d.