II.B.1.a.i. One-level two-channel case

For a simple one-level, two-channel case for which the shift factor is set to zero, the various cross sections defined in Section II.B.1.a can easily be expressed in terms of resonance parameters. Users are reminded that SAMMY is by no means restricted to this simple case and can be used with as many levels and as many channels as are needed to describe the particular physical situation. Nevertheless, it is useful to examine the cross section equations for this simple case: while these equations are a crude over-simplification for most physical situations, there is often physical insight to be gained by examination of these equations.

For this simple case, the X matrix of Eq. (II B1.4) takes the form

$$\begin{aligned} X &= \sqrt{P}L^{-1}\left(L^{-1}-R\right)^{-1}R\sqrt{P} \\ &= \begin{bmatrix} \dfrac{\sqrt{P_1}}{iP_1} & 0 \\ 0 & \dfrac{\sqrt{P_2}}{iP_2} \end{bmatrix} \begin{bmatrix} \dfrac{1}{iP_1}-\dfrac{\gamma_1^2}{D} & -\dfrac{\gamma_1\gamma_2}{D} \\ -\dfrac{\gamma_1\gamma_2}{D} & \dfrac{1}{iP_2}-\dfrac{\gamma_2^2}{D} \end{bmatrix}^{-1} \begin{bmatrix} \dfrac{\gamma_1^2}{D} & \dfrac{\gamma_1\gamma_2}{D} \\ \dfrac{\gamma_1\gamma_2}{D} & \dfrac{\gamma_2^2}{D} \end{bmatrix} \begin{bmatrix} \sqrt{P_1} & 0 \\ 0 & \sqrt{P_2} \end{bmatrix} , \end{aligned} \tag{II B1 ai.1}$$

in which the subscript on the penetrabilities denotes the channel number (not the angular momentum), the symbol D has been used for $E_\lambda - E - i\overline{\gamma}_{\lambda\gamma}^{\,2}$, and the subscript λ has been omitted from the reduced-width amplitudes for simplicity's sake. This equation can be rewritten as

$$X = \frac{iP_1P_2D}{iD} \begin{bmatrix} \dfrac{1}{\sqrt{P_1}} & 0 \\ 0 & \dfrac{1}{\sqrt{P_2}} \end{bmatrix} \begin{bmatrix} P_2\left(D-iP_1\gamma_1^2\right) & -iP_1P_2\gamma_1\gamma_2 \\ -iP_1P_2\gamma_1\gamma_2 & P_1\left(D-iP_2\gamma_2^2\right) \end{bmatrix}^{-1} \begin{bmatrix} \gamma_1^2 & \gamma_1\gamma_2 \\ \gamma_1\gamma_2 & \gamma_2^2 \end{bmatrix} \begin{bmatrix} \sqrt{P_1} & 0 \\ 0 & \sqrt{P_2} \end{bmatrix}$$

$$\begin{aligned} &= \frac{P_1P_2}{P_1P_2\left(D^2-iP_1\gamma_1^2D-iP_2\gamma_2^2D\right)} \begin{bmatrix} \dfrac{1}{\sqrt{P_1}} & 0 \\ 0 & \dfrac{1}{\sqrt{P_2}} \end{bmatrix} \begin{bmatrix} P_1\left(D-iP_2\gamma_2^2\right) & iP_1P_2\gamma_1\gamma_2 \\ iP_1P_2\gamma_1\gamma_2 & P_2\left(D-iP_1\gamma_1^2\right) \end{bmatrix} \\ &\qquad \times \begin{bmatrix} \gamma_1^2 & \gamma_1\gamma_2 \\ \gamma_1\gamma_2 & \gamma_2^2 \end{bmatrix} \begin{bmatrix} \sqrt{P_1} & 0 \\ 0 & \sqrt{P_2} \end{bmatrix} , \end{aligned}$$

or

$$X = \frac{1}{\left(D^2 - iP_1\gamma_1^2 D - iP_2\gamma_2^2 D\right)} \begin{bmatrix} \frac{1}{\sqrt{P_1}} & 0 \\ 0 & \frac{1}{\sqrt{P_2}} \end{bmatrix}$$

$$\times \begin{bmatrix} P_1\gamma_1^2 D - iP_1P_2\gamma_1^2\gamma_2^2 + iP_1P_2\gamma_1^2\gamma_2^2 & P_1 D\gamma_1\gamma_2 - iP_1P_2\gamma_1\gamma_2^3 + iP_1P_2\gamma_1\gamma_2^3 \\ iP_1P_2\gamma_1^3\gamma_2 + P_2 D\gamma_1\gamma_2 - iP_1P_2\gamma_1^3\gamma_2 & iP_1P_2\gamma_1^2\gamma_2^2 + P_2\gamma_2^2 D - iP_1P_2\gamma_1^2\gamma_2^2 \end{bmatrix} \begin{bmatrix} \sqrt{P_1} & 0 \\ 0 & \sqrt{P_2} \end{bmatrix}$$

$$= \frac{1}{\left(D^2 - iP_1\gamma_1^2 D - iP_2\gamma_2^2 D\right)} \begin{bmatrix} \frac{1}{\sqrt{P_1}} & 0 \\ 0 & \frac{1}{\sqrt{P_2}} \end{bmatrix} \begin{bmatrix} P_1\gamma_1^2 D & P_1 D\gamma_1\gamma_2 \\ P_2 D\gamma_1\gamma_2 & P_2\gamma_2^2 D \end{bmatrix} \begin{bmatrix} \sqrt{P_1} & 0 \\ 0 & \sqrt{P_2} \end{bmatrix}$$

$$= \frac{1}{\left(D - iP_1\gamma_1^2 - iP_2\gamma_2^2\right)} \begin{bmatrix} P_1\gamma_1^2 & \sqrt{P_1P_2}\gamma_1\gamma_2 \\ \sqrt{P_1P_2}\gamma_1\gamma_2 & P_2\gamma_2^2 \end{bmatrix} ,$$

or, finally,

$$\begin{aligned} X &= \frac{1}{\left(E_\lambda - E - i\overline{\gamma}_\gamma^2 - iP_1\gamma_1^2 - iP_2\gamma_2^2\right)} \begin{bmatrix} P_1\gamma_1^2 & \sqrt{P_1P_2}\gamma_1\gamma_2 \\ \sqrt{P_1P_2}\gamma_1\gamma_2 & P_2\gamma_2^2 \end{bmatrix} \\ &= \frac{1}{\left(E_\lambda - E - i\Gamma/2\right)} \begin{bmatrix} \Gamma_1/2 & \sqrt{\Gamma_1\Gamma_2}/2 \\ \sqrt{\Gamma_1\Gamma_2}/2 & \Gamma_2/2 \end{bmatrix} , \end{aligned} \qquad \text{(II B1 ai.2)}$$

in which Γ is the sum of the partial widths $\Gamma_1 + \Gamma_2 + \Gamma_\gamma$.

In this form, X can be substituted into the equations for the various cross sections. Assuming the second channel is a reaction channel, Eq. (II B1 a.2) for the total cross section becomes

$$\begin{aligned} \sigma_{total}(E) &= \frac{4\pi}{k_\alpha^2} g_J \left[\sin^2\varphi_c + \frac{\Gamma\Gamma_1}{4d}\cos(2\varphi_c) - \frac{(E - E_\lambda)\Gamma_1}{2d}\sin(2\varphi_c) \right] \\ &= \frac{2\pi}{k_\alpha^2} g_J \left[1 - \left(1 - \frac{\Gamma\Gamma_1}{2d}\right)\cos(2\varphi_c) - \frac{(E - E_\lambda)\Gamma_1}{d}\sin(2\varphi_c) \right] , \end{aligned} \qquad \text{(II B1 ai.3)}$$

in which d has been used to represent $\left|\left(E_\lambda - E - i\Gamma/2\right)\right|^2 = \left(E - E_\lambda\right)^2 + \left(\Gamma/2\right)^2$. Similarly, the elastic cross section, Eq. (II B1 a.3), can be expressed as

$$\sigma_{elastic}(E) = \frac{4\pi}{k_\alpha^2} \sum_J g_{J\alpha} \sum_c \left[\sin^2 \varphi_c \left(1 - 2\frac{\Gamma\Gamma_1}{4d}\right) - \frac{(E-E_\lambda)\Gamma_1}{2d} \sin(2\varphi_c) + \left(\frac{\Gamma\Gamma_1}{4d}\right)^2 + \left(\frac{(E-E_\lambda)\Gamma_1}{2d}\right)^2 \right] \quad , \tag{II B1 ai.4}$$

which reduces to

$$\sigma_{elastic}(E) = \frac{2\pi}{k_\alpha^2} \sum_J g_{J\alpha} \sum_c \left[1 - \cos 2\varphi_c \left(1 - \frac{\Gamma\Gamma_1}{2d}\right) - \sin 2\varphi_c \frac{(E-E_\lambda)\Gamma_1}{d} - \frac{\Gamma_1(\Gamma_\gamma + \Gamma_2)}{2d} \right] \quad . \tag{II B1 ai.5}$$

The reaction cross section, Eq. (II B1 a.4), becomes

$$\begin{aligned} \sigma_{reaction}(E) &= \frac{4\pi}{k_\alpha^2} \; g \left[\left(\frac{\Gamma\sqrt{\Gamma_1\Gamma_2}}{4d} \right)^2 + \left(\frac{(E-E_\lambda)\sqrt{\Gamma_1\Gamma_2}}{2d} \right)^2 \right] \\ &= \frac{\pi g}{k_\alpha^2} \left[\frac{\Gamma_1\Gamma_2}{d} \right] \quad , \end{aligned} \tag{II B1 ai.6}$$

and, finally, the capture cross section, Eq. (II B1 a.6), is

$$\begin{aligned} \sigma_{capture}(E) &= \frac{4\pi g}{k_\alpha^2} \left[\frac{\Gamma\Gamma_1}{4d} - \left\{ \left(\frac{\Gamma\Gamma_1}{4d}\right)^2 + \left(\frac{(E-E_\lambda)\Gamma_1}{2d}\right)^2 + \left(\frac{\Gamma\sqrt{\Gamma_1\Gamma_2}}{4d}\right)^2 + \left(\frac{(E-E_\lambda)\sqrt{\Gamma_1\Gamma_2}}{2d}\right)^2 \right\} \right] \\ &= \frac{4\pi g}{k_\alpha^2} \left[\frac{\Gamma\Gamma_1}{4d} - \left\{ \frac{\Gamma_1^{\;2}}{4d} + \frac{\Gamma_1\Gamma_2}{4d} \right\} \right] = \frac{\pi g}{k_\alpha^2} \left[\frac{\Gamma_1 \overline{\Gamma}_\gamma}{d} \right] \quad . \end{aligned} \tag{II B1 ai.7}$$