III.E.4. Paramagnetic Cross Section

The total cross section of nuclides like erbium, holmium, and thulium at low neutron energies may include both nuclear and paramagnetic cross sections. The paramagnetic cross section may be written in the following form [YD98]:

$$\sigma_{PM} = C\left(\frac{\left(A\tan^{-1}\left(BE^{P} \right) \right)}{BE^{P}\left(1 + \left(BE^{P} \right)^{2} \right)^{4}} \right)^{2} \quad . \qquad \text{(III E4.1)}$$

Here E is the energy in eV, and A, B, P, and C are parameters (variables) whose default values are given in Table III E4.1; these may be treated either as constants or as varied parameters whose values are to be determined by the Bayes (generalized least-squares) fitting procedure.

As presently implemented within SAMMY, the paramagnetic cross section may be invoked only in calculations for data types “total cross section” or “transmission.” In particular, paramagnetic cross sections cannot be included in angular distributions nor in generating self-shielding or multiple-scattering corrections for capture or fission yields.

Table III E4.1. Default parameter values for paramagnetic cross sections

	A	*B*	*P*	*C*
Tm	6.1691	1.1308	0.3367	1.0
Er	7.9012	1.1278	0.3263	1.0
Ho	8.7597	1.1435	0.3225	1.0

Input for using PM cross section (for total cross sections only) is given in Table VI B.2, card set 12. See test case tr055 for examples of this input.

Although SAMMY can search on any or all of these parameters, the user should be aware that variables A and C are mathematically redundant; it is only the product CA^2 that is unique. When values for both parameters are permitted to vary, the correlation coefficient will always be -1.

Derivatives of σ_{PM} are as follows: Let x represent the quantity BE^{P}. The derivatives with respect to each of the four parameters are then

$$\frac{\partial\sigma}{\partial A} = \frac{2\sigma}{A} \quad , \qquad \text{(III E4.2)}$$

$$\frac{\partial\sigma}{\partial B} = C\,A^2\,2f\,\frac{d\,f}{d\,x}\,\frac{x}{B} \quad , \tag{III E4.3}$$

$$\frac{\partial\sigma}{\partial P} = C\,A^2\,f\,\frac{d\,f}{d\,x}\,x\ln\left(E\right) \quad , \tag{III E4.4}$$

and

$$\frac{\partial\sigma}{\partial C} = \frac{\sigma}{C} \quad . \tag{III E4.5}$$

In these equations we have made the substitution

$$f = \frac{\tan^{-1}x}{x\left(1+x^2\right)^4} \quad , \tag{III E4.6}$$

so that

$$\frac{d\,f}{d\,x} = \frac{1}{x\left(1+x^2\right)^5}\left[1-\frac{1-9x^2}{x}\tan^{-1}x\right] \quad . \tag{III E4.7}$$