II.C.2.b. Kinematics for elastic scattering

In the case of elastic scattering, primed quantities are exactly equal to unprimed, and the Q-value is zero. The equations of Section II.C.2 therefore simplify to the forms shown here.

The initial momentum K in the center-of-mass (COM) system is found from Eq. (II C2.3) to be

$$K = \hbar k = \frac{M}{m+M}\sqrt{2mE} \quad , \tag{II C2 b.1}$$

and the final COM momentum K' is found in Eq. (II C2.4) to be

$$K' = \frac{M}{m+M}\sqrt{2mE} \quad . \tag{II C2 b.2}$$

The laboratory energy of the outgoing particle is found from Eqs. (II C2.5) and (II C2.6) to be

$$E' = \left[\frac{1}{m+M}\left\{m\,\mu + \sqrt{M^2 - m^2\left(1-\mu^2\right)}\right\}\right]^2 E \quad , \tag{II C2 b.3}$$

with $\mu = \cos\theta$.

The relationships between the COM and lab angles, Eqs. (II C2.7) and (II C2.8), become

$$\nu = -\frac{m}{M}\left(1-\mu^2\right) + \mu\sqrt{1-\left(\frac{m}{M}\right)^2\left(1-\mu^2\right)} \tag{II C2 b.4}$$

and

$$\mu = \frac{M\nu + m}{\sqrt{M^2 + m^2 + 2mM\nu}} \quad , \tag{II C2 b.5}$$

where $\nu = \cos\beta$ and β is the COM angle. The derivative of ν with respect to μ, Eq. (II C2.9), is

$$\frac{d\nu}{d\mu} = \frac{\left(\mu m + \sqrt{M^2 - m^2\left(1-\mu^2\right)}\right)^2}{\sqrt{M^2 - m^2\left(1-\mu^2\right)}} \quad . \tag{II C2 b.6}$$