II.C.4.a. Charged-particle initial states

To derive the equations for the angle-differential cross sections for charged-particle incident channels, we begin with the Lane and Thomas [AL58] expression [page 292, Eq. (2.6)]. When this expression is corrected (for a missing complex conjugate, a missing minus sign, and missing delta functions), summed over the exit channel spins s', and averaged over the incident channel spins *s*, the resulting equation for the angle-differential cross section is

$$\begin{aligned} \frac{d\sigma_{\alpha\alpha'}}{d\Omega_{CM}} &= \sum_L B_{L\alpha\alpha'}(E)\, P_L(\cos\beta) + \frac{\pi}{k_\alpha^2}\left| C_\alpha(\beta)\right|^2 \delta_{\alpha\alpha'} \\ &+ \frac{\sqrt{4\pi}}{k_\alpha^2}\sum_{Jsl} g_J \,\mathrm{Re}\left[-i\left(\frac{e^{2iw_{l\alpha}} - U_{cc}}{2}\right) C_\alpha^* \; P_l(\cos\beta)\right]\delta_{\alpha\alpha'} \quad . \end{aligned} \tag{II C4 a.1}$$

Here we have again used the convention that $c = \{\alpha, l, s, J\}$. For the charged-particle case, the definition of $B_{L\alpha\alpha'}(E)$ is modified slightly from the non-Coulomb case [Eq. (II B1 b.2)] to give

$$\begin{aligned} B_{L\alpha\alpha'}(E) &= \frac{1}{4k_\alpha^2}\sum_{J_1}\sum_{J_2}\sum_{l_1 s_1}\sum_{l'_1 s'_1}\sum_{l_2 s_2}\sum_{l'_2 s'_2} \\ &\times G_{\{l_1 s_1 l'_1 s'_1 J_1\}\{l_2 s_2 l'_2 s'_2 J_2\} L}\ \frac{1}{(2i+1)(2I+1)} \\ &\times \mathrm{Re}\left[(e^{2iw_{l\alpha}}\delta_{c_1 c'_1} - U_{c_1 c'_1})\,(e^{-2iw_{l\alpha}}\delta_{c_2 c'_2} - U^*_{c_2 c'_2})\right] \quad . \end{aligned} \tag{II C4 a.2}$$

In the final line of Eq. (II C4 a.2), the quantity c_1 is substituted for the expression $\{\alpha, l_1, s_1, J_1\}$, c_2 for $\{\alpha, l_2, s_2, J_2\}$, c'_1 for $\{\alpha', l'_1, s'_1, J_1\}$, and c'_2 for $\{\alpha', l'_2, s'_2, J_2\}$. The geometric term *G* in Eq. (II C4 a.2) is the same as for the non-Coulomb case and is defined in Eqs. (II B1 b.3) to (II B1 b.10). Notation for summation indices is the same as in the non-Coulomb case.

What is different here is the presence of the exponential involving the Coulomb phase-shift difference $w_{l\alpha}$, defined in Eq. (II C4.7). Also, the scattering matrix contains the $w_{l\alpha}$ in the definition of Ω; the Sommerfeld parameter η_α in Eq. (II C4.1) is defined as

$$\eta_\alpha = \frac{z\,Z\,e^2 \mu_\alpha}{\hbar^2 k_\alpha} \quad . \tag{II C4 a.3}$$

The additional terms in Eq. (II C4 a.1) involve the function C_α, which is defined as

$$C_\alpha = \frac{1}{\sqrt{4\pi}} \eta_\alpha \ \operatorname{cosec}^2\left(\frac{\beta}{2}\right) e^{-2i\eta_\alpha \ \ln\left[\sin\left(\frac{\beta}{2}\right)\right]} \quad . \tag{II C4 a.4}$$

It is this term which is infinite at $\beta = 0$ (forward scattering) and which causes the (angle-integrated) elastic-scattering cross section to be infinite.

Center-of-mass vs Laboratory

Angular distribution cross sections are sometimes reported as if measured in the center-of-mass system rather than in the laboratory system; hence, SAMMY can calculate either version. To specify which is wanted, insert one of the phrases

```
USE CENTER OF MASS Cross sections
USE LABORATORY CROSS sections
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into your INPut file (see Tables VI A.1 and VI A1.2). Center-of-mass is the default.

Acknowledgements

The author is grateful to SAMMY users Jeff Blackmon and Olivier Bouland for uncovering bugs in early implementations of charged-particle elastic scattering, and to Eric Berthoumieux (author of the ANARKI R-matrix code) for providing calculations to assist in debugging this portion of SAMMY.