II.C.1.a. Quantum vector algebra

For a complete description of sum rules for quantum vectors, the reader is referred to textbooks on elementary quantum mechanics. Here we simply state the vector sum rules without detailed explanation.

Let $\vec{a}$ be a quantized vector. The value of this vector, generally written either *a* or $\left|\vec{a}\right|$, is either a positive half-integer or a positive integer. That is to say, *a* can have any of the values 0, 1/2, 1, 3/2, 2, 5/2, etc. For example, the spin of a neutron or proton is 1/2, and the spin of an alpha particle is 0. The orbital angular momentum *l* for a pair of particles is integral, *l* = 0, 1, 2, 3, etc.

Given two quantized vectors $\vec{a}$ and $\vec{b}$, and let $\vec{c}=\vec{a}+\vec{b}$ be the sum of the two vectors. The possible values for *c* are then

$$\left|a-b\right| \le c \le a+b \quad , \tag{II C1 a.1}$$

where the allowed values of c are separated by one unit. Examples are shown in Table II C1 a.1. Values of *a* and *b* are in the left-most column and the uppermost row; values for *c* are in the other cells of the table. Because Eq. (II C1 a.1) is symmetric with respect to *a* and *b*, entries are made only in the lower triangular half of the table.

Each spin vector has an associated parity, which can be positive or negative. For example, protons, neutrons, and alpha particles have positive parity; many nuclides have negative parity. The parity associated with angular momentum *l* is $(-1)^l$. Parity is conserved when two vectors are added; the product of the parities of the two components is the parity of the resulting vector. A vector which is formed as the sum of two positive-parity vectors will have positive parity, a vector which is formed as the sum of two negative-parity vectors will have positive parity, and a vector which is formed as the sum of one positive-parity vector and one negative-parity vector will have negative parity. In other words, if *a* and *b* have the same parity, *c* has positive parity. If f *a* and *b* have different parity, *c* has negative parity.

Table II C1 a.1. Allowed values for the sum of two quantized vectors

	0	½	1	3/2	2	5/2	3	7/2	4	9/2
0	0									
1/2	1/2	0, 1								
1	1	1/2, 3/2	0, 1, 2							
3/2	3/2	1, 2	1/2, 3/2, 5/2	0, 1, 2, 3						
2	2	3/2, 5/2	1, 2, 3	1/2, 3/2, 5/2, 7/2	0, 1, 2, 3, 4					
5/2	5/2	2, 3	3/2, 5/2, 7/2	1, 2, 3, 4	1/2, 3/2, 5/2, 7/2, 9/2	0, 1, 2, 3, 4, 5				
3	3	5/2, 7/2	2, 3, 4	3/2, 5/2, 7/2, 9/2	1, 2, 3, 4, 5	1/2, 3/2, 5/2, 7/2, 9/2, 11/2	0, 1, 2, 3, 4, 5, 6			
7/2	7/2	3, 4	5/2, 7/2, 9/2	2, 3, 4, 5	3/2, 5/2, 7/2, 9/2, 11/2	1, 2, 3, 4, 5, 6	1/2, 3/2, 5/2, 7/2, 9/2, 11/2, 13/2	0, 1, 2, 3, 4, 5, 6, 7		
4	4	7/2, 9/2	3, 4, 5	5/2, 7/2, 9/2, 11/2	2, 3, 4, 5, 6	3/2, 5/2, 7/2, 9/2, 11/2, 13/2	1, 2, 3, 4, 5, 6, 7	1/2, 3/2, 5/2, 7/2, 9/2, 11/2, 13/2, 15/2	0, 1, 2, 3, 4, 5, 6, 7, 8	
9/2	9/2	4, 5	7/2, 9/2, 11/2	3, 4, 5, 6	5/2, 7/2, 9/2, 11/2, 13/2	2, 3, 4, 5, 6, 7	3/2, 5/2, 7/2, 9/2, 11/2, 13/2, 15/2	1, 2, 3, 4, 5, 6, 7, 8	1/2, 3/2, 5/2, 7/2, 9/2, 11/2, 13/2, 15/2, 17/2	0, 1, 2, 3, 4, 5, 6, 7, 8, 9