## II.C.4. Modifications for Charged Particles

The penetrabilities $P_l$, shift factors $S_l$, and potential-scattering phase shifts $\varphi_l$ defined in Table II A.1 apply only to non-Coulomb interactions such as those involving incident neutrons. Often, however, the two particles in a channel will both have a positive charge; examples are the exit channels for (n,α) or (n,p) interactions, and the incident channels in the inverse (reciprocal) measurements (α,n) and (p,n). In this case the expressions for penetrabilities, shift factors, and phase shifts must be modified to include the long-range interaction; see, for example, the discussion of Lane and Thomas [AL58].

An extension for SAMMY to include Coulomb penetrabilities, shift factors, and phase shifts was developed by R. O. Sayer [RS00] (and modified by the SAMMY author) and used first for analysis of $^{16}$O data [LL98, RS00]. FORTRAN routines used for this purpose are a modified version of the routine COULFG of Barnett [AB82].

Additional changes were required to calculate the cross sections for incident charged particles; details are given in Section II.C.4.a. Because the Coulomb interaction is long range, only the angle-differential cross sections are calculable; the angle-integrated cross sections are infinite.

Expressions for $P_l$, $S_l$, and $\varphi_l$ for particle pair $\alpha$ involve the parameter $\eta_\alpha$, which is defined as

$$\eta_\alpha = \frac{z_\alpha Z_\alpha e^2 \mu_\alpha}{\hbar^2 k_\alpha} \quad , \tag{II C4.1}$$

where $z$ and $Z$ are the charge numbers for the two particles in the particle pair. The reduced mass $\mu_\alpha$ is defined in the usual manner as

$$\mu_\alpha = \frac{m_\alpha M_\alpha}{m_\alpha + M_\alpha} \quad , \tag{II C4.2}$$

where $m_\alpha$ and $M_\alpha$ are the masses of the two particles in channel $\alpha$. The center-of-mass (COM) momentum $\hbar k_\alpha$ is defined in the same manner as in Eq. (II C2 a.8), as

$$\hbar^2 k_\alpha^2 = \frac{2 m_\alpha M_\alpha}{\left(m_\alpha + M_\alpha\right)} \frac{M}{\left(m + M\right)} \left(E - \Xi\right) \quad . \tag{II C4.3}$$

In Eq. (II C4.3), the masses of particles in the incident channel are denoted without subscripts; these masses may be different from the masses in particle pair $\alpha$. If the excitation energy is given in the COM system, this expression takes the form

$$\hbar^2 k_\alpha^2 = \frac{2 m_\alpha M_\alpha}{\left(m_\alpha + M_\alpha\right)} \frac{M}{\left(m + M\right)} \left( E - \left[ \frac{m + M}{M} \right] \Xi_{\text{com}} \right) \quad . \tag{II C4.4}$$

In the SAMMY input, the user can specify the value of the excitation energy either in the laboratory system [as in Eq. (II C4.3)] or in the COM system; SAMMY will make the appropriate conversions. The default is laboratory. Users who wish to provide COM values should include the phrase "CM COULOMB EXCITATION ENERGIES" as needed.

The reaction Q-value is the negative of variable $\Xi_{COM}$ = ECHAN (in the COM system) in card set 10.1 or 10.2 of Table VI A.1. When using the particle-pair input, card set 4 or 4a of Table VIA.1, it is possible to give the Q-value directly; this is the recommended input format.

Recall that a channel is defined by the particle pair (with mass, spin, and charge for each of the two particles, plus the Q-value) plus the spin quantum numbers *l*, *s*, and *J*. Quantities defined above in Eqs. (II C4.1) through (II C4.4) depend only on the particle pair α and not on the spin quantum numbers. Other quantities (below) depend also on the value of *l*.

If $a_c$ is the channel radius for this channel, we again define $\rho$ as

$$\rho = k_\alpha a_c \qquad \text{(II C4.5)}$$

The penetrabilities $P_l(\eta,\rho)$, shift factors $S_l(\eta,\rho)$, and phase shifts $\varphi_l(\eta,\rho)$ are then calculated as functions of $F_l(\eta,\rho)$ and $G_l(\eta,\rho)$, the regular and irregular Coulomb wave functions, respectively. The equations are as follows:

$$P_l = \frac{\rho}{A_l^2}, \qquad S_l = \frac{\rho}{A_l}\frac{\partial A_l}{\partial \rho}, \qquad \text{and} \quad \cos\varphi_l = \frac{G_l}{A_l} \quad , \qquad \text{(II C4.6)}$$

where

$$A_l^2 = F_l^2 + G_l^2 \quad . \qquad \text{(II C4.7)}$$

In Eqs. (II A.1) and (II A.5), the Coulomb phase-shift difference $w_c$ is required for charged-particle interactions. From Lane and Thomas [AL58], this quantity has the value

$$w_c = \begin{cases} 0 & l = 0 \\ \sum_{n=1}^{l} \tan^{-1}\left(\frac{\eta_\alpha}{n}\right) & l \neq 0 \end{cases} \quad . \qquad \text{(II C4.8)}$$

Finally, we note that an alternative version of the Coulomb functions is available in SAMMY. This alternative, modified from files provided by Hale [GH02], requires longer run time but appears to be more accurate at low values of $\rho$ (and corresponding high values of $\eta$). SAMMY will automatically switch to the more accurate version when it discerns the need. To invoke this alternative for all calculations, include the phrase

```
USE ALTERNATIVE COULomb functions
```

in the INPut file.