II.C.5. Inverse Reactions (Reciprocity)

Occasionally a user may wish to include data from an inverse reaction in the same evaluation as the forward reaction. For example, for an evaluation of the ^{16}O resonance parameters, Sayer [RS00] wanted to include $^{16}O(n,\alpha)^{13}C$ data. No such data existed, but $^{13}C(\alpha,n)^{16}O$ data were available.

Unfortunately SAMMY does not have the capability of including reciprocal data in the same evaluation (using the same resonance parameters). SAMMY was designed with the intent of treating one incident particle (originally a neutron) and many different types of nuclides within the target. Other codes (e.g., EDA [GH75]) were designed with a different philosophy: to simultaneously treat all interactions leading to the same compound nucleus. Eventually the SAMMY author hopes to add similar capabilities to the SAMMY code.

Meanwhile, two alternatives are available: (1) The SAMMY user can convert the data using reciprocal relationships, and include the converted data within his or her evaluation. (2) If there is no need for simultaneous* fitting, resonance parameter values can be converted to those appropriate for the reciprocal reaction. Either of these two can be accomplished by application of the principle of detailed balance.

To convert the cross section from the $A'\left(a',a\right)A$ reaction to the $A\left(a,a'\right)A'$ reaction, we first consider the center-of-mass (COM) system, in which the energies are easily related by

$$E_{COM} = E'_{COM} - Q \quad . \tag{II C5.1}$$

Elementary kinematics (as illustrated in Figure II C5.1) gives the conversion to the laboratory values E and E',

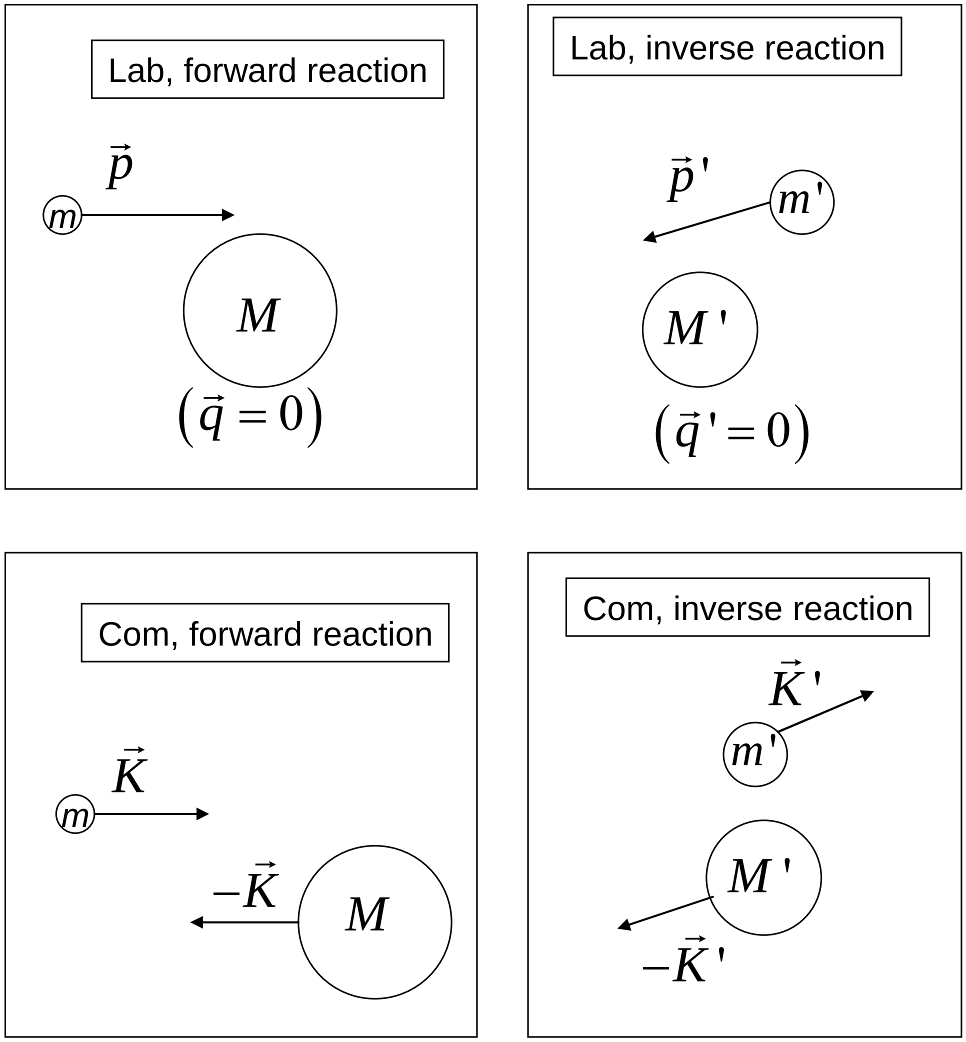
$$E_{COM} = \frac{M}{m+M} E \quad , \qquad E'_{COM} = \frac{M'}{m'+M'} E' \quad , \qquad Q = -\frac{M}{m+M} \Xi_{lab} \quad , \tag{II C5.2}$$

and algebra then gives

$$E' = \left(E - \Xi_{lab}\right) \frac{M}{m+M} \frac{m'+M'}{M'} \quad . \tag{II C5.3}$$

* By "simultaneous" is meant either (1) truly simultaneous or (2) "sequential using the covariance matrix from one SAMMY run fitting one data set as input to another run fitting another data set." See Section IV of this manual for details of both possibilities.

Figure II C5.1. Schematic of kinematics for inverse reactions.



The R-matrix for the $A'(a',a)A$ system must have the same value (at comparable energies) as the R-matrix for the $A(a,a')A'$ system. Hence

$$
\begin{aligned}
R'_{cc"} &= \sum_\lambda \frac{\gamma'_{\lambda c}\gamma'_{\lambda c"}}{E'_\lambda - E' - i\Gamma'_{\lambda\gamma}/2} \\
&= \sum_\lambda \frac{\gamma'_{\lambda c}\gamma'_{\lambda c"}}{E'_\lambda - \left[\left(E-\Xi_{lab}\right)\dfrac{M}{m+M}\dfrac{m'+M'}{M'}\right] - i\Gamma'_{\lambda\gamma}/2} = \sum_\lambda \frac{\gamma_{\lambda c}\gamma_{\lambda c"}}{E_\lambda - E - i\Gamma_{\lambda\gamma}/2} = R_{cc"} \quad .
\end{aligned}
\tag{II C5.4}
$$

In order for the equality on the bottom line of Eq. (II C5.4) to hold, the unprimed resonance parameters must be defined as follows:

$$E_\lambda = E'_\lambda\, q + \Xi_{lab} \ , \qquad \Gamma_{\lambda c} = \Gamma'_{\lambda c}\, q \ , \qquad \gamma_{\lambda c} = \gamma'_{\lambda c} \sqrt{q} \ , \tag{II C5.5}$$

where

$$q = \left[\frac{m+M}{M}\right]\left[\frac{M'}{m'+M'}\right] \tag{II C5.6}$$

(for c = any channel, for example neutron, fission, or capture). These equations may be used to convert the resonance parameters.

To convert the experimental data, recall that there is a multiplicative kinematic factor of $1/K^2$, where K is the momentum of the incident particle in the COM frame. For $A'(a',a)A$, this term is

$$\frac{1}{K'^2} = \frac{(m'+M')^2}{2m'(M')^2 E'} \ , \tag{II C5.7}$$

and for $A(a,a')A'$, the term is

$$\frac{1}{K^2} = \frac{(m+M)^2}{2mM^2E} \ . \tag{II C5.8}$$

The experimental cross sections must be multiplied by the ratio of these two values, and appropriate energy substitutions made.

Another multiplicative factor that must be adjusted is the spin statistical factor, which also reflects the parameters of the incident channel. Since the compound nuclear spin J is the same in either system, the correct multiplier is the ratio of the two:

$$\frac{g_{J:A(aa')A'}}{g_{J:A'(a'a)A}} = \frac{(2i'+1)(2I'+1)}{(2i+1)(2I+1)} \ . \tag{II C5.9}$$

With these changes, the cross section for the $A(a,a')A'$ reaction may be written in terms of the cross section for the $A'(a',a)A$ reaction as

$$\sigma_{A(aa')A'}(E) = \frac{(2i'+1)(2I+1)}{(2i+1)(2I+1)} \frac{m'M'^2}{mM^2} \frac{(m+M)^2}{(m'+M')^2} \frac{E'}{E} \sigma_{A'(a'a)A}(E') \ . \tag{II C5.10}$$